MOHID-GLM : code developments for 3D waves-current interactions

Implementation of the glm2-RANS equations by Ardhuin et al. 2008

MOHIDing workshop – 7-8 June, 2018 – Lisbon, Portugal

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Motivations

Need for 3D wave-current implementation

- 3D features in the flow may be generated by multiple factors
  - Density stratification by continental freshwater outflows
  - Wind-induced circulation
  - Vertical shear in rip currents

- Surface gravity waves have a large impact on nearshore circulation in energetic environments
  - Wave setup
  - Longshore currents
  - Rip currents
  - Horiztonal & vertical mixing
  - Etc.

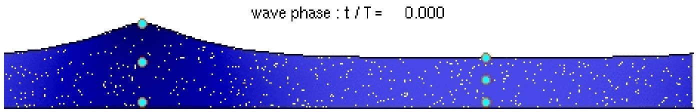
→ **Need for 3D models including the effect of waves**

Issue

The difficulty of including waves in phase-averaged 3D models

- Decomposition of flow components

$$\mathbf{u} = \overbrace{\langle \mathbf{u} \rangle}^{\text{vitesse moyenne}} + \overbrace{\tilde{\mathbf{u}}}^{\text{vitesse oscillante}} + \overbrace{\mathbf{u}'}^{\text{vitesse turbulente}} \qquad \mathbf{m}^T = \overbrace{\mathbf{m}}^{\text{qdm du courant sous-jacent}} + \overbrace{\mathbf{m}^W}^{\text{qdm résiduelle des vagues}}$$

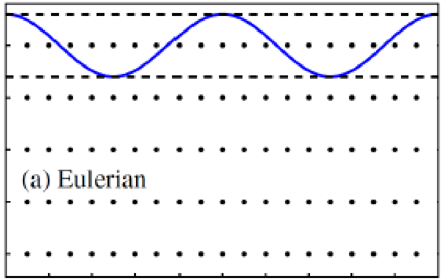


→ Objective = representing interactions between mean flow and oscillating flow (momentum, mass, energy)

- Problem: it is difficult to apply an Eulerian average on phases in 3D

$$\overline{\phi(\mathbf{x}, t)} = \frac{1}{T} \int_{-T/2}^{T/2} \phi(\mathbf{x}, t + t')\mathrm{d}t'$$

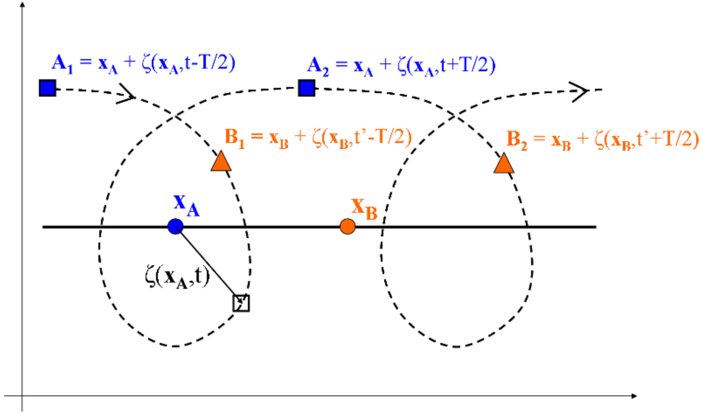
$$\Rightarrow \frac{1}{T} \int_{-T/2}^{T/2} \phi(\mathbf{x} + \xi, t + t')\mathrm{d}t'$$



The Generalized Lagrangian Mean approach

Main principle

- Derive new equations using a Lagrangian coordinate change, from the mean position X to the instantaneous position X + ξ



→ Allows to « follow » the oscillatory mouvement

→ Generalized Lagrangian Mean (Andrews & McIntyre, 1978)

$$\overline{\phi(x,t)}^L = \overline{\phi(x+\zeta(x,t),t)}$$

→ New mean advection velocity + new Lagrangian derivation operator

$$\overline{\mathbf{u}(\mathbf{x},t)}^L$$

→ Application of the GLM to the RANS equations

$$\overline{D}^L\phi = \frac{\partial\phi}{\partial t} + \overline{u}_i^L\frac{\partial\phi}{\partial x_i}$$

- Ardhuin et al. 2008 : asymptotic formulation of GLM equations for surface gravity waves → Development order 2
  - Small wave steepness
  - Limited vertical shear of the mean current
  - Slowly varying propagation environment

The Generalized Lagrangian Mean approach

GLM2-RANS equations

- Momentum equation

$$\frac{\partial u}{\partial t}+\frac{\partial u\underline{u}}{\partial x}+\frac{\partial v\underline{u}}{\partial y}+\frac{\partial w\underline{u}}{\partial z}-f\underline{v}\frac{1}{\rho}\frac{\partial p^H}{\partial x}=fV_S+\frac{\partial \underline{u}}{\partial x}U_S+\frac{\partial \underline{v}}{\partial x}V_S-\frac{\partial J}{\partial x}+F_{m,x}+F_{d,x}$$

where: u is now the GLM velocity (= tracer advection velocity)

U_S, V_S = Stokes drift components

$\underline{u} = u - U_S$ = quasi-Eulerian current

p^H = hydrostatic pressure

J = wave-induced pressure (barotropic)

$F_{m,x}$ = momentum flux from the turbulent mixing

$F_{d,x}$ = momentum flux induced by wave breaking

f = Coriolis parameter

The Generalized Lagrangian Mean approach

GLM2-RANS equations

- Wave induced pressure $$J = g\frac{kE}{\sinh(2kD)}$$

- 3D Stokes drift $$(U_S, V_S) = \sigma k(\cos\theta, \sin\theta)E\frac{\cosh(2kz + 2kh)}{\sinh^2(kD)}$$

- Wave-breaking induced flux of momentum: $$(\tau_{oc,x}, \tau_{oc,y}) = \rho_w \cdot \int \frac{g}{C(f,\theta)}(\cos\theta, \sin\theta)S_{oc}(f,\theta)dfd\theta$$

→ **Given by the wave model as an input to MOHID**

The Generalized Lagrangian Mean approach

GLM2-RANS equations

- Mass equation

$$\frac{\partial \underline{\eta}}{\partial t} + \frac{\partial D(\overline{\underline{u}} + \overline{U_s})}{\partial x} + \frac{\partial D(\overline{\underline{v}} + \overline{V_s})}{\partial y} = 0$$

where: u is now the GLM velocity (= tracer advection velocity)

U_S, V_S = Stokes drift components

$\underline{\eta}$ mean free surface elevation

D total water depth

upper bar = vertical integration

The Generalized Lagrangian Mean approach

GLM2-RANS equations

- Surface boundary condition

$$\frac{\partial \eta}{\partial t}+(\underline{u}+U_s)\frac{\partial \eta}{\partial x}+(\underline{v}+V_s)\frac{\partial \eta}{\partial y}=\underline{w}+W_S \qquad K_M\frac{\partial \underline{u}_\alpha}{\partial z}=\tau_{a,\alpha}-\tau_{aw,\alpha}$$

- Bottom boundary condition

$$\underline{u}\frac{\partial h}{\partial x}+\underline{v}\frac{\partial h}{\partial y}=\underline{w} \qquad K_M\frac{\partial \underline{u}_\alpha}{\partial t}=\rho C_D\left|\underline{u}\right|\underline{u}_\alpha+\tau_{cw,\alpha}$$

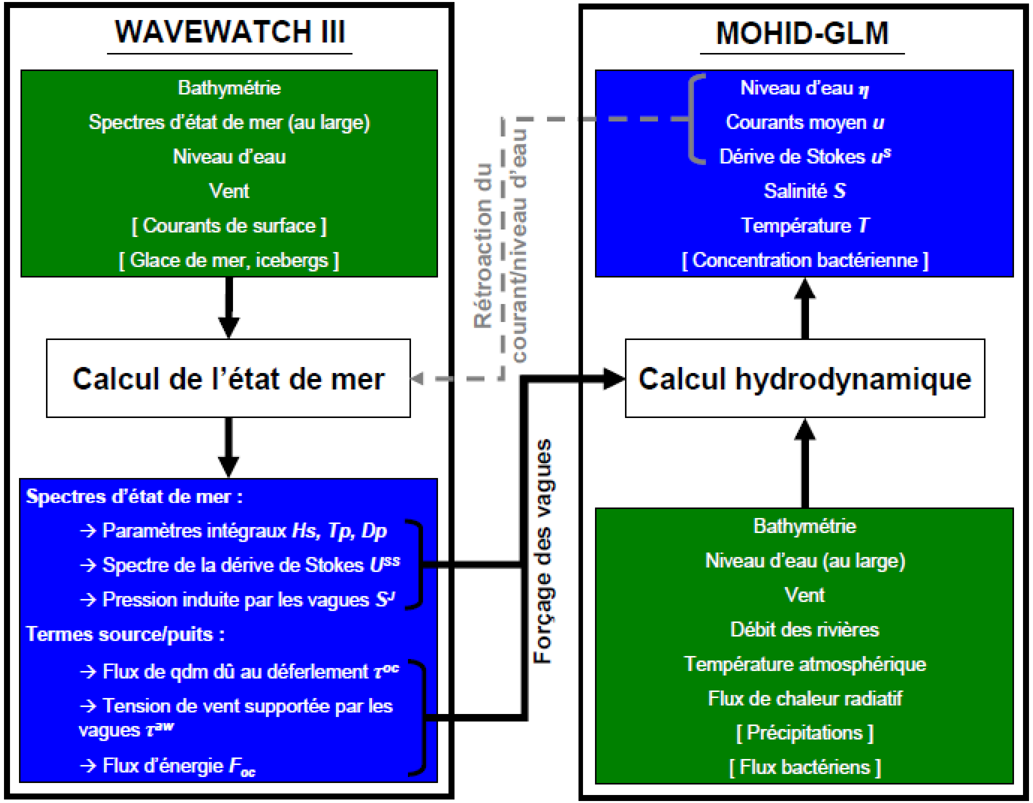
With $\underline{w}$ the vertical component of the quasi-Eulerian velocity

W_S the vertical components of the Stokes drift (given by the wave model)

$\tau_{a,\alpha}$ = total wind stress

$\tau_{aw,\alpha}$ = wind stress supported by waves (given by the wave model)

Implementation of MOHID-GLM



Implementation of MOHID-GLM

MOHID Water modifications

- Conservative formulation
- Reading of additional forcing variables *(Module Waves)*
- Compute Us, Vs from the frequency spectrum of the Stokes Drift *(Module Waves)*
- Add **Us, Vs** in advection terms *(Module Hydrodynamic)*
- Add contribution of **J and Fd** in the momentum equation *(Module Hydrodynamic)*
- Modify **boundary conditions** *(Modules Hydrodynamic, InterfaceWaterAir, InterfaceSedimentWater)*
- Modify surface boundary conditions for **TKE equation** in the k-epsilon model *(Modules GOTM and InterfaceWaterAir)*

→ For detailed discretization see: Delpey 2012.

Mass conservation:

$$\frac{\partial \eta}{\partial t} = -\frac{\partial}{\partial x_\alpha} \overbrace{\left(\int_{-h}^{\eta} (u_\alpha + \overline{u}_\alpha^S) \mathrm{dz} \right)}^{\mathcal{A}_1}.$$

Horizonal momentum conservation:

$$\frac{\partial u_\alpha}{\partial t} + \overbrace{\frac{\partial \left[u_\alpha (u_\beta + \overline{u}_\beta^S) \right]}{\partial x_\beta} + \frac{\partial \left[u_\alpha (w + \overline{w}^S) \right]}{\partial z}}^{\mathcal{A}_2}$$

$$= -\frac{1}{\rho_0} \frac{\partial p^a}{\partial x_\alpha} + (-g + b(\eta)) \frac{\partial \eta}{\partial x_\alpha} + \int_z^{\eta} \frac{\partial b}{\partial x_\alpha} \mathrm{dz} - \overbrace{\frac{\partial S^J}{\partial x_\alpha}}^{\mathcal{B}} + \overbrace{\overline{u}_\beta^S \frac{\partial u_\beta}{\partial x_\alpha}}^{\mathcal{C}}$$

$$+ \frac{\partial}{\partial x_\beta} \left(K_H \frac{\partial u_\alpha}{\partial x_\beta} \right) + \frac{\partial}{\partial z} \left(K_V \frac{\partial u_\alpha}{\partial z} \right).$$

Vertical advection velocity:

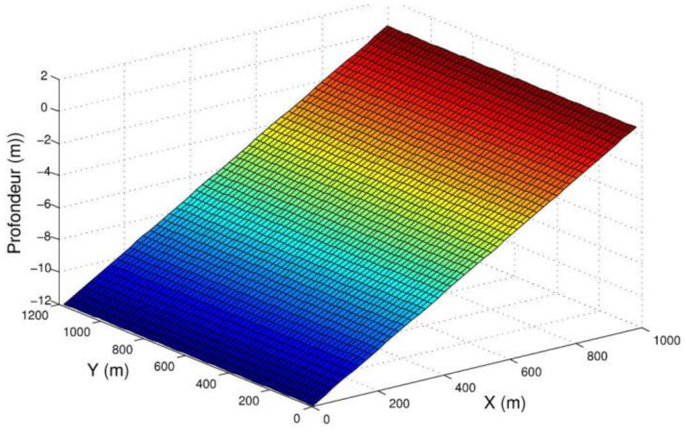
$$\overline{w}^L = w + \overline{w}^S = -\frac{\partial}{\partial x_\alpha} \overbrace{\left(\int_{-h}^{z} (u_\alpha + \overline{u}_\alpha^S) \mathrm{dz} \right)}^{\mathcal{A}_3}.$$

Tracer equation:

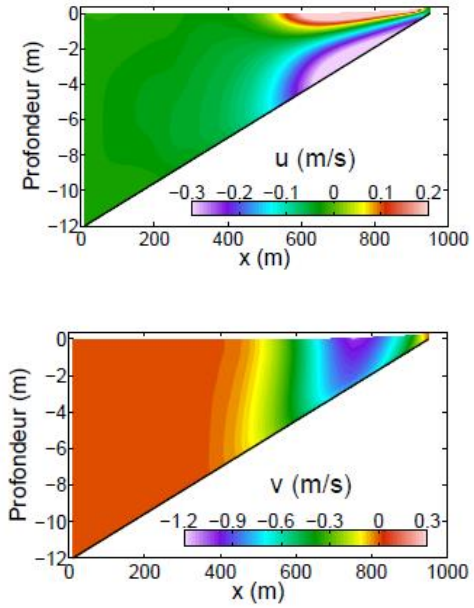
$$\frac{\partial C}{\partial t} + \overbrace{\frac{\partial \left[(u_\alpha + \overline{u}_\alpha^S) C \right]}{\partial x_\alpha} + \frac{\partial \left[(w + \overline{w}^S) C \right]}{\partial z}}^{\mathcal{A}_4} = \frac{\partial}{\partial x_\alpha} \left(\frac{K_H}{\sigma_C} \frac{\partial C}{\partial x_\alpha} \right) + \frac{\partial}{\partial z} \left(K_T \frac{\partial C}{\partial z} \right) + S_C.$$

Validation of MOHID GLM

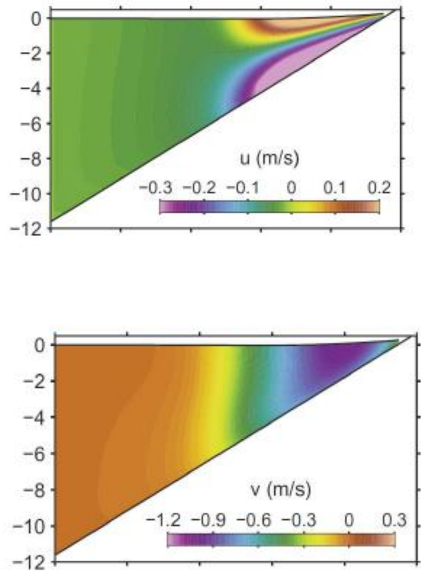
Haas & Warner 2009 case



- Solution de MOHID-GLM ($z_{0,s} = 0.2H_S$)



- Solution de ROMS [Uchiyama et al., 2010]



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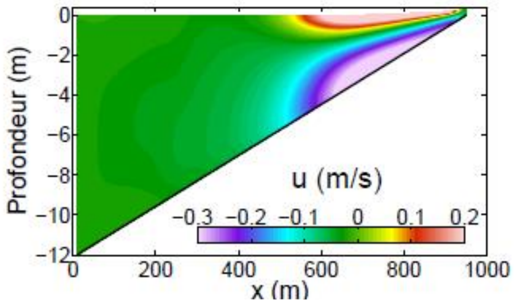

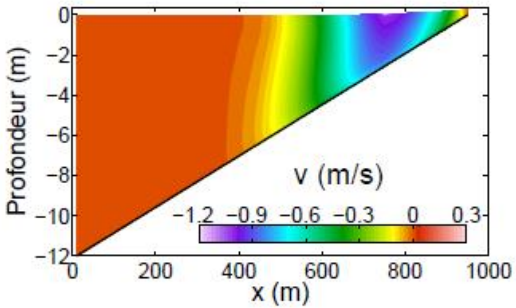

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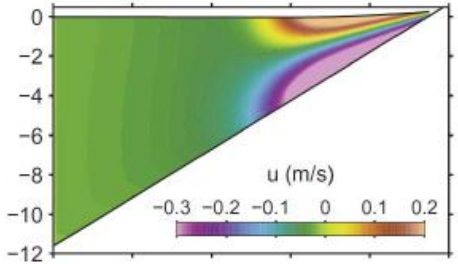
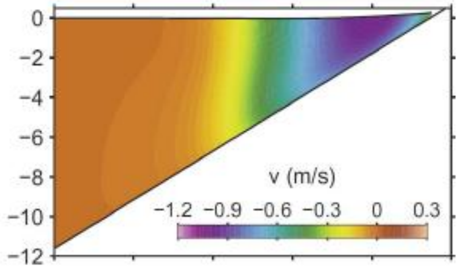
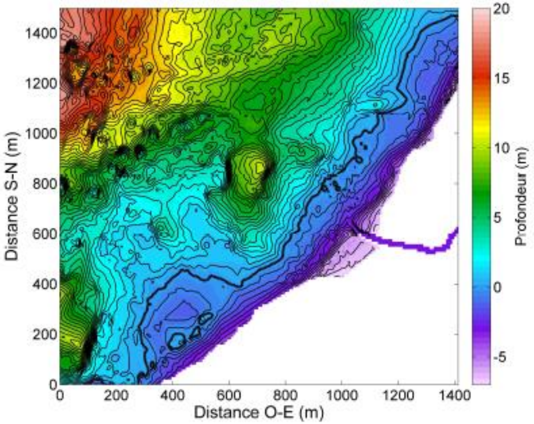
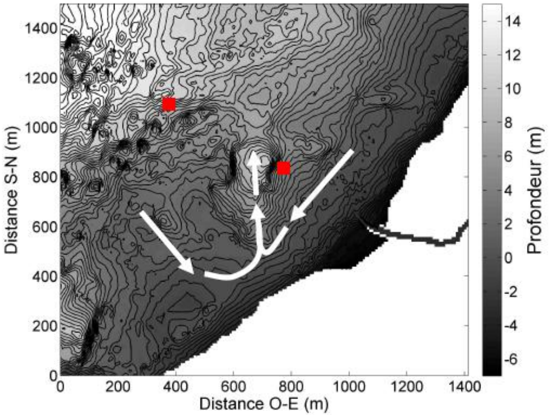




Real case application

Application to the Uhabia beach, SW France





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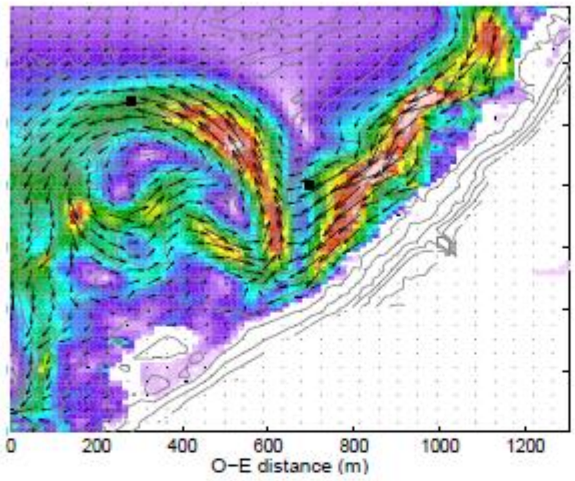

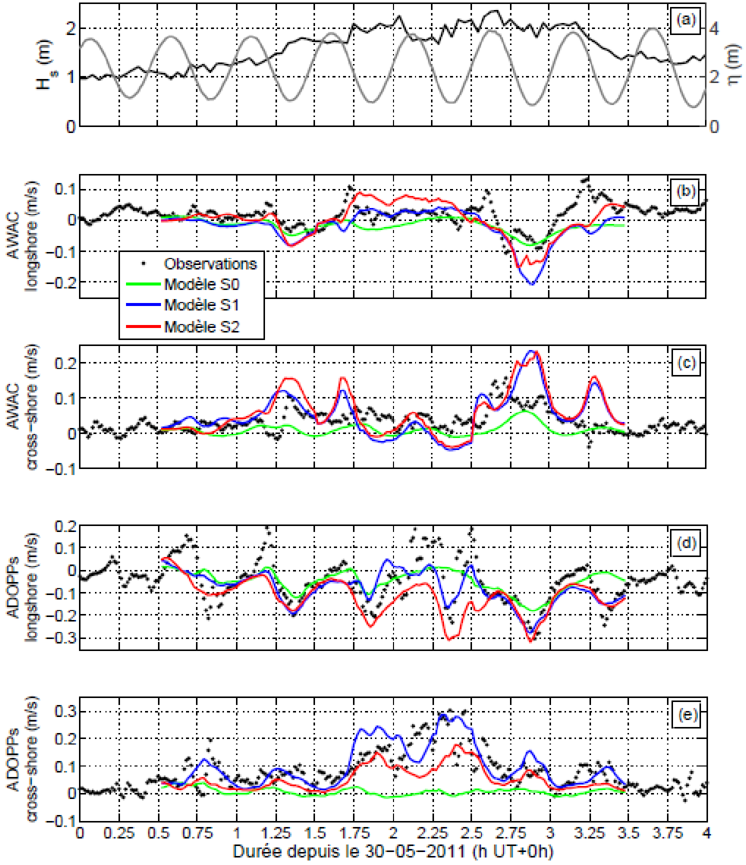

Real case application

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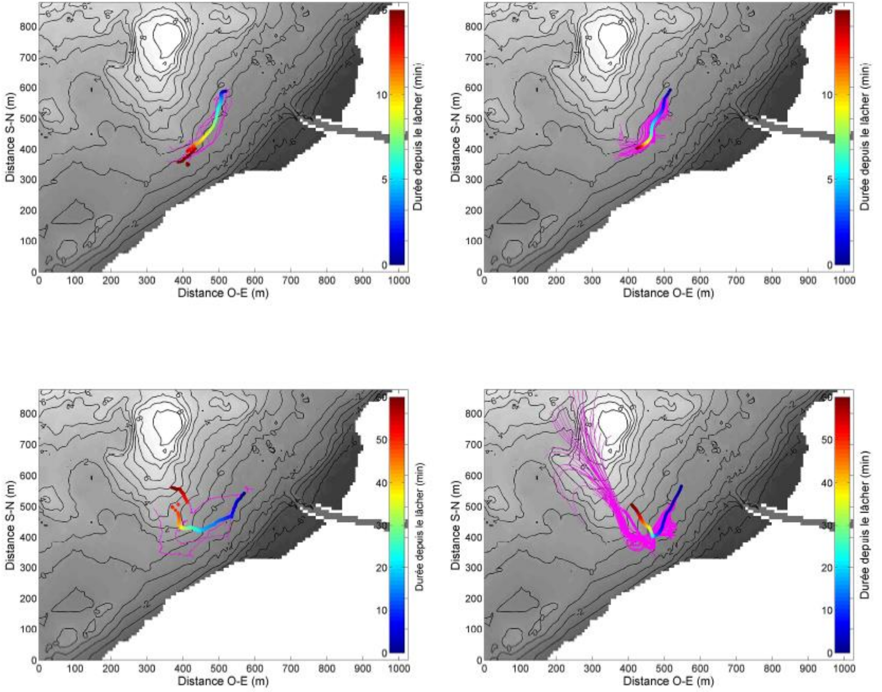
Drifter observation | Model



A (very) simplified formulation

Exponential vertical decrease of radiation stress

$$S_{ij}'(x,y,z) = \frac{ke^{kz}}{e^{k\eta} - e^{-kh}} S_{ij}(x,y)$$

k = wave number, h = depth, η = free surface elevation

- No theoritical justification → « Empirical »
- Principle: consistant with barotropic balance + having a vertical distribution « a bit more plausible » than using homogeous vertical distribution of wave radiation stree
- Allows to produce some features like undertow → See e.g. Franz et al. 2017

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